

Closing today: HW_8A, 8B (8.3, 9.1)

Closing Next Wed: HW_9A, 9B (9.3, 9.4)

Final: Sat, Dec 9th, 1:30-4:20 in KANE 130

9.3: Separable Differential Equations

Entry Task: (Motivation)

Implicitly differentiate $x^2 + y^3 = 8$

and solve for $\frac{dy}{dx}$.

Idea: separate and integrate both sides.

Entry Task continued:

Find the *explicit* solution for $\frac{dy}{dx} = \frac{-2x}{3y^2}$

with $y(0) = 2$.

9.3: Separable Differential Equations

A **separable** differential equation is one that can be written as:

$$\frac{dy}{dx} = f(x)g(y).$$

(or $\frac{dy}{dx} = \frac{f(x)}{g(y)}$ or $\frac{dy}{dx} = \frac{g(y)}{f(x)}$.)

Example: Find the explicit solution to

$$\frac{dy}{dx} = \frac{x}{y^4} \text{ with } y(0) = 1.$$

Example: Find the explicit solution to

$$\frac{dy}{dx} = \frac{x \sin(2x)}{3y} \quad \text{with } y(0) = -1.$$

Example: Find the explicit solution to

$$(x + 1) \frac{dy}{dx} = \frac{x^2}{e^y} \quad \text{with } y(0) = 0.$$

Law of Natural Growth

Assumption: “*The rate of growth of a population is proportional to the size of the population.*”

$P(t)$ = population at year t ,

$\frac{dP}{dt}$ = rate of change of the
population

The law of natural growth assumes

$$\frac{dP}{dt} = kP,$$

for some constant k

(we call k the relative growth rate).

Find the explicit solution to

$$\frac{dP}{dt} = kP \text{ with } P(0) = P_0$$

1. 500 bacteria are in a dish at $t=0$ hr.
8000 bacteria are in the dish at $t=3$ hr.
Assume the population grows at a
rate proportional to its size.
Find the function, $B(t)$, for the
bacteria population with respect to
time.

2. The *half-life* of cesium-137 is 30 years. Suppose we start with a 100-mg sample. The mass decays at a rate proportional to its size.

Find the function, $m(t)$, for the mass with respect to time.